Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 1 September 2006)

Chapter 8

Q8.1 $\bar{\delta} = 2.66\,(m_{\rm D}/m_{\rm Saturn})^{-2/3}$ and $R = 1.587 \times 10^{-3}\,(m_{\rm D}/m_{\rm Saturn})^{-1/3}\,e$. This gives $m_{\rm D}/m_{\rm Saturn} = 0.0101$; hence $\bar{\delta} = 56.8$ and $R = 0.0352$. This leads to an implausible density of 7832 g cm^{-3}.

Q8.2 Let M be the mass of the star; use primed quantities to denote the outer mass, unprimed quantities for the inner mass. The resulting amplitudes in the eccentricities are: $\Delta e = 4.87m'/M$, $\Delta e' = 3.50m/M$ for pair (1,2); $\Delta e = 7980m'/M$, $\Delta e' = 2280m/M$ for pair (1,3); $\Delta e = 4.87m'/M$, $\Delta e' = 3.62m/M$ for pair (2,3). Note the large amplitudes for the (1,3) pair and make a comparison with the LONGSTOP Uranus experience.

Q8.3 Taking the semi-major axis of Jupiter from Table A.2, the first-order resonances in the range are the 2:1 at 3.27791 AU (width 0.1568 AU), 3:2 at 3.97091 AU (width 0.2728 AU), 4:3 at 4.29528 AU (width 0.3634 AU) and 5:4 at 4.48412 AU (width 0.4394 AU). The second-order resonances in the range are the 3:1 at 2.50152 AU (width 0.02872 AU), 5:3 at 3.70156 AU (width 0.1209 AU), 7:5 at 4.15782 AU (width 0.2231 AU) and 9:7 at 4.40069 AU (width 0.3285 AU). Using only these resonances there is overlap between two or more resonances in the semi-major axis range of 4.046 AU $< a <$ 4.565 AU. However, this is a slightly artificial upper limit because including the remaining resonances leads to more overlap.

Q8.4 Taking the semi-major axes of Jupiter and Saturn from Table A.2, the jovian external first-order resonances in the range are the 1:2 at 8.2598 AU (width 0.5930 AU), 2:3 at 6.81833 AU (width 0.5938 AU), 3:4 at 6.30343 AU (width 0.6311 AU) and 4:5 at 6.03797 AU (width 0.6742 AU). The jovian external second-order resonances in the range are the 3:5 at 7.31448 AU (width 0.2044 AU), 5:7 at 6.51183 AU (width 0.2567 AU), 7:9 at 6.51244 AU (width 0.3131 AU), and 9:11 at 5.94818 AU (width 0.3710 AU). The saturnian internal first-order resonances are the 2:1 at 6.00798 AU (width 0.1574 AU), 3:2 at 7.27815 AU (width 0.2737 AU), 4:3 at 7.87268 AU (width 0.3647 AU), 5:4 at 8.2188 AU (width 0.4409 AU), 6:5 at 8.44554 AU (width 0.5075 AU), 7:6 at 8.60565 AU (width 0.5672 AU) and 8:7 at 8.72476 AU (width 0.6217 AU). The saturnian internal second-order resonances are the 5:3 at 6.78447 AU (width 0.1213 AU), 7:5 at 7.62073 AU (width 0.2239 AU), 9:7 at 8.06588 AU (width 0.3287 AU), 11:9 at 8.34286 AU (width 0.4368 AU), 13:11 at 8.53194 AU (width 0.5446 AU), 15:13 at 8.66928 AU (width 0.6529 AU) and 17:15 at 8.7736 AU (width 0.7614 AU). Using only these resonances there is overlap between two or more resonances in the semi-major axis ranges of 5.763 AU $< a <$ 6.640 AU, 6.724 AU $< a <$ 6.845 AU, 7.212 AU $< a <$ 7.415 AU, 7.690 AU $< a <$ 7.733 AU and 7.901 AU $< a <$ 9.036 AU. Again, the last upper limit is slightly misleading because including the remaining resonances leads to more overlap.

Q8.5 The particle's eccentricity at the start of the evolution should be taken to be zero (see error listing). The times of encounter and the increases in e at each possible e- and e^2-resonance are: 13:11 ($t = 3.56 \times 10^5$ y, $\Delta e = 0.00162$), 6:5 ($t = 5.40 \times 10^6$ y, $\Delta e = 0.00742$), 11:9 ($t = 1.14 \times 10^7$ y, $\Delta e = 0.000144$), 5:4 ($t = 1.85 \times 10^7$ y, $\Delta e = 0.00144$), 9:7 ($t = 2.73 \times 10^7$ y, $\Delta e = 0.000124$), 4:3 ($t = 3.82 \times 10^7$ y, $\Delta e = 0.00141$), 7:5 ($t = 5.22 \times 10^7$ y, $\Delta e = 0.000110$), 3:2 ($t = 7.10 \times 10^7$ y, $\Delta e = 0.00161$), 5:3 ($t = 9.72 \times 10^7$ y, $\Delta e = 0.0000991$), 2:1 ($t = 1.37 \times 10^8$ y, $\Delta e = 0.00236$) and 3:1 ($t = 2.02 \times 10^8$ y, $\Delta e = 0.00299$). Note that when $e_{\rm init} \ll e_{\rm crit}$ the formula $e_{\rm init}^2 + e_{\rm final}^2 = e_{\rm crit}^2$ can be used. This corresponds to putting $\delta_{\rm t} = 0$ in Eqs. (8.218) and (8.232).

Q8.6 Note that the changes in semi-major axis are not required (see error listing). The variations in e and ϖ for each planet should have a period of 5.56 y. The amplitudes are $\Delta e_1 = 0.000284$, $\Delta e_2 = 0.000369$, $\Delta\varpi_1 = 0.739^\circ$ and $\Delta\varpi_2 = 1.06^\circ$. Remember that these are only due to the effects of the resonance.