Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 1 September 2006)

Chapter 4

Q4.1 You should be able to show that there is no contribution to J_2 from a sphere of uniform density and so only the thin shell contributes.

Q4.2 (a) $x = [(5\alpha\rho/2 - \rho_\mathrm{m})/(\rho - \rho_\mathrm{m})]^{1/2}$; $\rho_\mathrm{c} = \rho_\mathrm{m} + (\rho - \rho_\mathrm{m})^{5/2}/(5\alpha\rho/2 - \rho_\mathrm{m})^{3/2}$. (b) For Earth the relationships give $\rho_\mathrm{m} \leq 4.58$ g cm^{-3}, $\rho_\mathrm{c} \geq 7.30$ g cm^{-3}, $x \leq 0.91$. When we use R_c =3,480 km we have $\rho_\mathrm{m} = 4.18$ g cm^{-3} and $\rho_\mathrm{c} = 12.3$ g cm^{-3}.

Q4.3 (a) $\tan\epsilon = \omega\beta/(\omega_0^2 - \omega^2)$; $A = F/\left[(\omega_0^2 - \omega^2)^2 + \omega^2\beta^2\right]^{1/2}$. (b) To do the integral you must set $x = A\cos(\omega t - \epsilon)$ and $\dot{x} = -\omega A\sin(\omega t - \epsilon)$. (c) $E_\mathrm{max} = \frac{1}{2}\omega_0^2 A^2$ and hence $Q = \omega_0^2/(\beta\omega)$. This is often referred to as a "frequency-dependent Q" in geophysics, and implies $\epsilon \propto \omega$ for small phase lags and slow forcing.

Q4.4 The numerically-derived answers for the Moon's initial and final synchronous states under tidal evolution are: (a) [initial state] $a = 2.26R$, $P_\mathrm{orb} = P = 4.8$ h at time $\tau = -1.65 \times 10^9$y (for $Q = 12$). (b) [final state] $a = 77.5R$, $P_\mathrm{orb} = P = 39.7$ d at time $\tau = +6.7 \times 10^9$y. (c) [final state, with no solar torques] $a = 86.9R$, $P_\mathrm{orb} = P = 47.0$ d at time $\tau = +16.1 \times 10^9$y. Note that the above are averages of ~ 12 independent calculations, and may be accurate to $\pm 0.5\%$. The results for (a) and (c) may also be derived approximately from angular momentum conservation arguments.

Q4.5 For the first part you should make use of the fact that satellite tides cannot appreciably alter the orbital angular momentum, L, but only the z-component is conserved if $I \neq 0$ because the orbit will precess. The inclination damping timescale is $\tau_I = (2/3)\,(m_\mathrm{s}/m_\mathrm{p})\,(a/R_\mathrm{s})^5\,(Q/k_2)_\mathrm{s}\,(\sin I/\sin\epsilon)^2\,(n\cos I)^{-1}$. Make use of Eq. (4.198) and Eq. (4.156) to show that $\tau_I/\tau_e = 7\,(\sin I/\sin\epsilon)^2\,(\cos I)^{-1}$.

Q4.6 The theory behind this method is covered in Sect. 4.13 with the initial semi-major axis set to the synchronous value. The resulting lower limits for Mars, Jupiter, Saturn, Uranus and Neptune are 42, 1.12×10^6, 8.1×10^4, 7.9×10^4 and 5.4×10^4 where we have used a synchronous semi-major axis of two planetary radii for the case of Proteus.