

The Origin and Evolution of *Solar System Dynamics*

Carl Murray

*Astronomy Unit
Queen Mary University of London*

Stan Dermott Special Session, DDA, University of Chicago, June 24, 2026

Origin

Carl Murray at Queen Mary



Credit: Carl Murray

c.1978

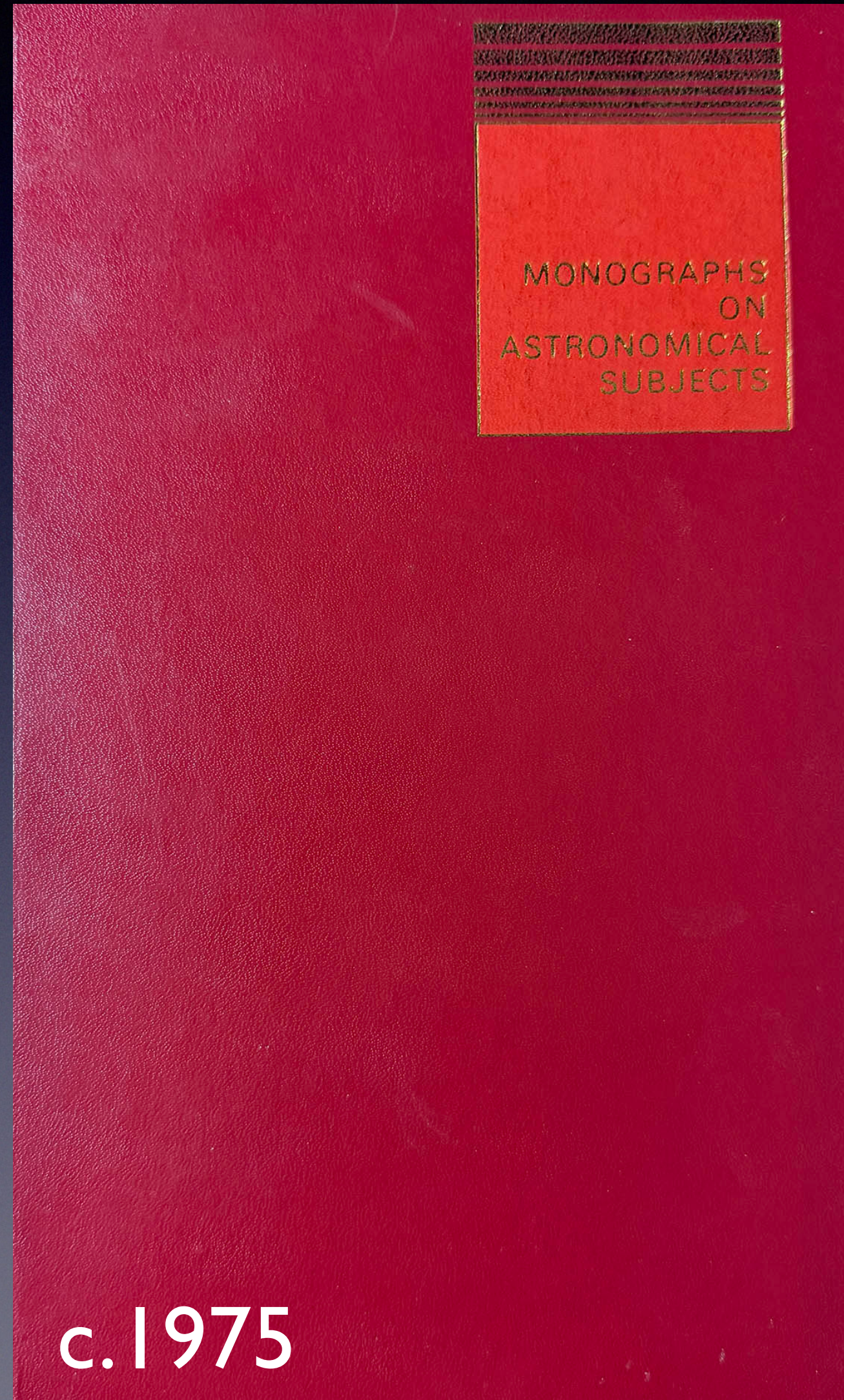
Stan Dermott at Cornell



Credit: Carl Murray

c.1982

Monographs published by Adams Hilger (IoP)



c.1975

Origin

To be published in the same series:

Magnetohydrodynamics

T. G. Cowling, M.A., D.Phil., F.R.S.

Theories of Gravitation and Geophysics

P. S. Wesson, Ph.D.

Interstellar Clouds

D. McNally, Ph.D.

X-ray and Gamma-Ray Astronomy

David Adams, Ph.D.

The Interpretation of Solar Flares

P. A. Sweet, Ph.D.

Solar X-ray Astronomy

J. H. Parkinson, Ph.D.

Solar Radio Emission

J. F. Vesecky, Ph.D.

Planetary Atmospheres

G. E. Hunt, Ph.D.

Orbital Motion

A. E. Roy, Ph.D.

Resonances in the Solar System

S. F. Dermott, Ph.D.

Resonances in the Solar System
S. F. Dermott, Ph.D.

“Dear Stan, ... I wondered how much progress you have made on your monograph?”

“Dear Stan, ... do you have any idea how much longer it will take to finish your monograph?”

“Dear Stan, ... **I am retiring soon.** Is there any chance that you can finish your monograph?”

(paraphrasing)

Origin



Our definition of solar system dynamics:

“The application of the techniques of celestial mechanics to solve real problems in planetary science”

Guiding Principles

- “Write about what you know”
- “Don’t write the text-book until you have given the lecture course”
- Write a book that we would have like to have read when we were starting out as researchers
- Write to convey understanding
- Write a book that is as self-contained as possible

Evolution

- Work started in 1982 before Carl left Cornell
- Annual visits to Ithaca (then Gainesville) and London
- Exchange of material by mail, FAX and email
- Working drafts of the book used at QM and UF
- Submitted to CUP in 1997

Structure

	v
<i>Preface</i>	xi
1 Structure of the Solar System	1
1.1 Introduction	1
1.2 The belief in number	2
1.3 Kepler's laws of planetary motion	3
1.4 Newton's universal law of gravitation	4
1.5 The Titius-Bode 'law'	5
1.6 Resonance in the solar system	5
1.7 The planetary system	10
1.8 The Jupiter system	10
1.9 The Saturn system	12
1.10 The Uranus system	13
1.11 The Neptune system	15
1.12 The Pluto system	15
1.13 The asteroid belt	14
1.14 Comets, meteors and dust	14
1.15 The preference for commensurability	15
1.16 Recent developments	17
Exercise questions 1	19
2 The Two-Body Problem	22
2.1 Introduction	22
2.2 Equation of motion	22
2.3 Orbital position and velocity	25
2.4 The mean and eccentric anomalies	32
2.5 Elliptic expansions	37
2.6 The guiding centre approximation	42
2.7 Barycentric orbits	45
2.8 The orbit in space	48
2.9 Perturbed orbits	54
2.10 Hamiltonian formulation	57
Exercise questions 2	60
3 The Restricted Three-Body Problem	63
3.1 Introduction	63
3.2 Equations of motion	64
3.3 The Jacobi integral	68
3.4 The Tisserand relation	71
3.5 Lagrangian equilibrium points	73
3.6 Location of equilibrium points	77
3.7 Stability of equilibrium points	84
3.8 1 The colinear points	90
3.9 2 The triangular points	92

- 2.5 Elliptic expansions
- 2.6 The guiding centre approximation
- 2.7 Barycentric orbits
- 2.10 Hamiltonian formulation

vi	
3.8 Motion near L_4 and L_5	94
3.9 Tadpole and horseshoe orbits	96
3.10 Orbits and zero velocity curves	102
3.11 Trojan asteroids and satellites	107
3.12 Janus and Epimetheus	110
3.13 Hill's equations	115
3.14 The effects of drag	121
3.15 1 Analysis of the Jacobi constant	122
3.16 2 Linear stability of the L_4 and L_5 points	123
3.17 3 Internal drag forces	126
Exercise questions 3	127
4 Tides, Rotation and Shape	130
4.1 Introduction	130
4.2 The tidal bulge	131
4.3 Potential theory	135
4.4 Tidal deformation	140
4.5 Rotational deformation	149
4.6 The Darwin-Radau relation	153
4.7 Shapes and internal structures of satellites	155
4.8 The Roche lobe	158
4.9 Tidal torques	160
4.10 Satellite tides	166
4.11 Tidal heating of Io	174
4.12 Tides on Titan	175
4.13 Tidal evolution	178
4.14 The double synchronous state	183
Exercise questions 4	186
5 Spin-Orbit Coupling	189
5.1 Introduction	189
5.2 Tidal despinning	189
5.3 The permanent quadrupole moment	194
5.4 Spin-orbit resonance	200
5.5 Capture into resonance	209
5.6 Forced librations	215
5.7 Surface of section	217
Exercise questions 5	222
6 The Disturbing Function	225
6.1 Introduction	225
6.2 The disturbing function	226
6.3 Expansion using Legendre polynomials	228

- 3.9 Tadpole and horseshoe orbits
- 3.12 Janus and Epimetheus
- 3.13 Hill's equations

	vii
6.4 Literal expansion in orbital elements	233
6.5 Literal expansion to second order	238
6.6 Terms associated with a specific argument	246
6.7 Use of the disturbing function	248
6.8 Lagrange's planetary equations	251
6.9 Classification of arguments in the disturbing function	253
6.10 1 Secular terms	254
6.11 2 Resonant terms	257
6.12 3 Short period and small amplitude terms	260
6.13 Sample calculations of the averaged disturbing function	261
6.14 1 Terms associated with the 3:1 commensurability	261
6.15 2 Terms associated with the 18:7 commensurability	263
6.16 The effect of planetary oblateness	265
Exercise questions 6	270
7 Secular Perturbations	274
7.1 Introduction	274
7.2 Secular perturbations for two planets	274
7.3 Jupiter and Saturn	279
7.4 Free and forced elements	283
7.5 Jupiter, Saturn and a test particle	289
7.6 Gauss's averaging method	293
7.7 Generalised secular perturbations	299
7.8 Secular theory for the solar system	302
7.9 Generalised free and forced elements	307
7.10 Hirayama families and the IRAS dust bands	309
7.11 Secular resonance	313
7.12 Higher order secular theory	317
Exercise questions 7	317
8 Resonant Perturbations	321
8.1 Introduction	321
8.2 The geometry of resonance	322
8.3 The physics of resonance	326
8.4 Variation of orbital elements	328
8.5 Resonance in the circular restricted three-body problem	332
8.6 The pendulum model	334
8.7 Libration width	337
8.8 The Hamiltonian approach	341
8.9 1 The e and e' resonances	348
8.10 2 The e^2 , e'^2 , I^2 and I'^2 resonances	355
8.11 3 The e^3 and e'^3 resonances	357
8.12 4 The ee' and II' resonances	359

- 7.2 Secular perturbations for two planets
- 7.3 Jupiter and Saturn
- 7.6 Gauss's averaging method
- 7.7 Generalised free and forced elements

Structure

• 8.9 The 2:1 resonance	364
1 Exact resonance	365
2 Medium amplitude libration	365
3 Large amplitude libration	367
4 Apocentric libration	367
5 Internal circulation	367
6 External circulation	369
7 Other types of motion	369
8 Comparison with analytical theory	370
8.10 The 3:1 and 7:4 resonances	371
8.11 Additional resonances and resonance splitting	372
8.12 Resonant encounters	375
1 Encounters with first-order resonances	376
2 Encounters with second-order resonances	382
8.13 The dynamics of capture and evolution in resonance	385
8.14 Two-body resonances in the solar system	387
1 The Titan-Hyperion resonance	388
2 The Mimas-Tethys resonance	389
8.15 Resonant encounters in satellite systems	390
8.16 Three-body resonance	391
8.17 The Laplace resonance	399
8.18 Secular and resonant motion	399
8.19 LONGSTOP Uranus	402
8.20 Pulsar planets	405
Exercise questions 8	406
• 9.3 Regular and chaotic orbits	409
9.1 Introduction	409
9.2 Sensitive dependence on initial conditions	410
9.3 Regular and chaotic orbits	413
1 The Poincaré surface of section	413
2 Regular orbits	415
3 Chaotic orbits	417
4 The Lyapounov characteristic exponent	418
9.4 Chaos in the circular restricted problem	420
9.5 Algebraic mappings	428
1 The standard map	428
2 Resonance map	431
3 Encounter maps	436
4 N -body maps	440
9.6 Separatrices and the orbit of Hyperion	448
9.7 The rotation of Hyperion	452
9.8 The Kirkwood gaps	456

	ix
1 Resonant structure of the asteroid belt	457
2 The 3:1 resonance	460
3 Other resonances	465
9.9 The Neptune-Pluto system	466
9.10 The stability of the solar system	469
Exercise questions 9	471
• 10.3 Resonances in rings	474
10.1 Introduction	474
10.2 Planetary ring systems	475
1 The rings of Jupiter	476
2 The rings of Saturn	478
3 The rings of Uranus	479
4 The rings of Neptune	481
5 Rings and satellites	481
10.3 Resonances in rings	481
1 Perturbations in semi-major axis and corotation resonances	482
2 Perturbations in eccentricity and Lindblad resonances	485
3 Perturbations in inclination and vertical resonances	489
4 Locations of resonances	490
10.4 Density waves and bending waves	492
10.5 Narrow rings and sharp edges	495
1 Spreading timescales	495
2 Localised effects of satellite perturbations	498
3 Shepherd satellites and radial confinement	504
4 Eccentric and inclined rings	506
5 Embedded satellites and horseshoe orbits	510
10.6 The Encke gap and Pan	511
10.7 The F ring of Saturn	515
10.8 The Adams ring of Neptune	518
10.9 The evolution of rings	520
10.10 The Earth's dust ring	522
Exercise questions 10	524
• 10.10 The Earth's dust ring	522
A Appendix – Solar System Data	526
A.1 Introduction	526
A.2 Astronomical constants	526
A.3 Julian date	527
A.4 Orbital elements of the planets and their variation	528
A.5 Planets, satellites and rings	530
A.6 Asteroids, Centaurs, trans-neptunian objects and comets	535
B Appendix – Expansion of the Disturbing Function	539

x	
B.1 Introduction	539
References	557
Index	575

- Appendix B – Expansion of the Disturbing Function

Impediments

NATURE VOL. 312 6 DECEMBER 1984

ARTICLES

505

Origin of the Solar System dust bands discovered by IRAS

S. F. Dermott, P. D. Nicholson, J.

Center for Radiophysics and Space Research, Cornell University

Measurements of longitudinal variations in the brightness and the infrared astronomical satellite determine the orbital elements of the cometary and asteroidal models of the origin of these bands.

Nature **312**, 505 (1984)



Proper Elements

ARTICLES

Masses of the satellites of Uranus

S. F. Dermott & P. D. Nicholson

Center for Radiophysics and Space Research, Space Sciences Building, Cornell University

The dynamical theory used to obtain the masses of the uranian satellites contains a fundamental error. The masses can be derived from the precession rates, but allow for the fact that the eccentricities, inclinations and precession rates are secular perturbations.

Nature **319**, 115 (1986)



Secular Perturbations

LETTERS TO NATURE

Tidal effects of disconnected hydrocarbon seas on Titan

Stanley F. Dermott* & Carl Sagan†

* Department of Astronomy, PO Box 112055, University of Florida, Gainesville, Florida 32611-2055, USA

† Laboratory for Planetary Studies, Cornell University, Ithaca, New York 14853, USA

THERMODYNAMIC and photochemical arguments¹⁻⁴ suggest that Titan, the largest satellite of Saturn, has a deep ocean of liquid hydrocarbons. At visible wavelengths, Titan's surface is obscured by a thick stratospheric haze, but radar observations⁵⁻⁷ have revealed large regions of high surface reflectivity that are inconsistent with a global hydrocarbon ocean. Titan's surface has also been imaged at infrared wavelengths⁸⁻¹⁰, and the highest-resolution data (obtained by the Hubble Space Telescope) show clear variations in surface albedo and/or topography¹⁰. The natural interpretation of these observations is that Titan, like the Earth, has continents and oceans. But Titan's high orbital eccentricity poses a problem for this interpretation, as the effects of oceanic tidal friction would have circularized Titan's orbit for most configurations of oceans and continents^{1,11}. Here we argue that a more realistic topography, in which liquid hydrocarbons are confined to a number of disconnected seas or crater lakes, may satisfy both the dynamical and observational constraints.

238

Recent observations⁸⁻¹⁰ of the surface of Titan through windows in the infrared spectrum seem to confirm that the satellite is in synchronous rotation. In such a case, because the orbital eccentricity of Titan is high, Saturn raises two important tides on the satellite: a radial tide due to the variation of Titan's distance from Saturn and a librational tide due to the fact that Titan rotates at a uniform rate whereas its orbital motion is keplerian and non-uniform. Frictional damping of these tidal oscillations results in the circularization of Titan's eccentric orbit on a timescale determined by the energy dissipation rate or, equivalently, by Titan's tidal dissipation function, Q_T , which varies inversely with the dissipation rate. Assuming that tidal damping has operated over the age of the Solar System, the observation that Titan's eccentricity is still as high as 0.029 implies that Q_T is high (>200) and that the energy dissipation rate is low¹.

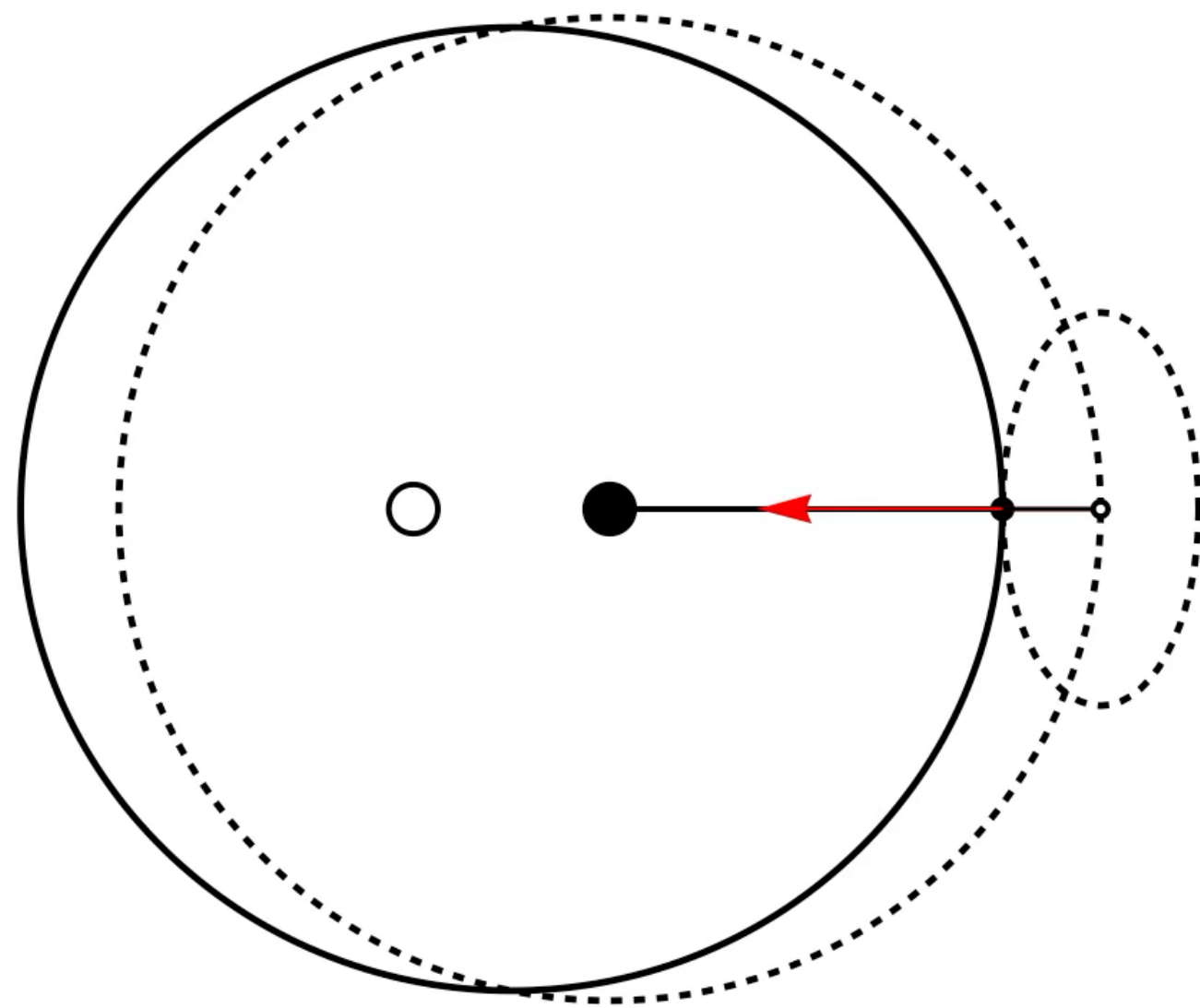
The Earth, which is the only body in the Solar System known to have an extensive ocean, has a tidal dissipation coefficient, Q_E , about 20 times smaller¹² than the lower limit on Q_T . This arises because land barriers modify the tidal flow and high rates of tidal dissipation occur in shallow seas and continental shelves (particularly the near-resonant shelves¹³) where the tidal currents are an order of magnitude greater than in the deep ocean¹²⁻¹⁴. The existence of both near-global oceans and extensive land barriers on Earth accounts for the low value of Q_E and led us to argue¹ that Titan's ocean must either be deep and global, with no significant protruding land, or else insignificant. The recent radar and infrared observations of Titan's surface

NATURE · VOL 374 · 16 MARCH 1995

Nature **374**, 238 (1995)

“Dermott’s Law”

Guiding Centre Approximation (Fig. 2.10)



- Stan discovered this empirically while a postdoc at Newcastle University (mid-1970s)
- Independently discovered by Danby (1988) in 2nd edition of his book (problem on p.136)

2. Show that if the square of the eccentricity of the Moon's orbit is neglected, then the Moon will always turn the same face toward the *empty* focus.

“A satellite in synchronous rotation keeps the same face pointed towards the empty focus of its orbit.”

Exercise Questions

Good News:

Typeset solutions for all problem questions exist!

Bad News:

They are in Norwegian!

The work was done in 2000 by Øystein Olsen, graduate student of Kaare Aksnes

Dermed så ser vi at den minste avstanden mellom Mars og Jorden varierer med en faktor på

$$\frac{0.67AU}{0.38AU} \approx 1.75$$

Ved å definere nære opposisjoner som opposisjoner der opposisjonen skjer når Mars er i nærheten av perihelion, så kan vi finne tidsrommet mellom to nære opposisjoner. Mars sin omløpstid er omtrent 687 dager. Hvis vi har en nær opposisjon ved et gitt tidspunkt, så vil Mars ved neste opposisjon ha passert perihelion for $780 - 687 = 93$ dager siden. Etter 7 eller 8 opposisjoner så er det 651 eller 744 dager siden Mars passerte perihelion, d.v.s. at Mars er omtrent ved perihelion igjen. Dermed vil det i snitt være ca $7.5 \cdot 780 \approx 5850$ dager mellom hver nære opposisjon. Dette er ca. 16 år.

På neste side følger et plott av avstanden mellom Jorden og Mars fra JD 2446545.0 til JD 2452545.0. Enheten på aksen er dager før/etter JD 2451545.0. Programmet som regnet ut dette og viser datoen for minste og største opposisjon, er skrevet i Mathematica og kan finnes på

Credit: Øystein Olsen

Miscellaneous

Celebrity Endorsement

Russian Translation



Ivan Shevchenko, 2009



Credit: Ashley Espy Kehoe

“... an amazing text-book – a bible for anyone entering the world of dynamical studies of Solar System bodies.”

Brian May, 2008

Future

- Material is being added to the SSD website at solarsystemdynamics.info
- At their last meeting in January 2024 Stan and Carl discussed plans for possible second edition
- Tentative plans to include von Zeipel-Kozai-Lidov mechanism, PPN, + ... (?) in second edition plus new chapter on drag forces and completely revised rings chapter
- No plans to include dynamics specific to exoplanetary systems

Thanks to the Error Finders:

Shafi Ahmed	Angela Flynn	Cesar Ocampo	Jean Teyssandier
Bill Andersen	Eric Ford	Gabriel Okša	Matt Tiscareno
Elsa Aristodemou	Tabare Gallardo	Mattia Orza	Scott Tremaine
Athos Athanasiou	Maria Genagritou	Sebastiano Padovan	Kleomenis Tsiganis
Mike Austin	Giacomo Giampieri	Pierre Paquette	Itai Tzur
Konstantin Batygin	Krzysztof Gozdziwski	Gabriele Pichierri	Harry Varvoglis
Bruce Bills	Atakan Gurkan	Elizabeth Platt	Dimitri Veras
Joe Burns	Doug Hamilton	Kees de Pooter	Paul Wiegert
Giammarco Campanella	Todd Humphreys	Than Putzig	Cong Yu
Robin Canup	Hiroshi Kinoshita	Darin Ragozzine	Shay Zucker
Apostolos Christou	Doug McNeil	Dmitry Savransky	
Josh Colwell	David Minton	Ivan Shevchenko	
Ricardo Reis Cordeiro	Helena Morais	Kedron Silsbee	
Paul Cresswell	Fathi Namouni	Amir Siraj	
Russell Deitrick	Greg Novak	Travis Stenborg	



Stanley F. Dermott (1942–2025)

Co-author of
Solar System Dynamics

Credit: Carl Murray