

## Answers and Hints to Exercise Questions in “Solar System Dynamics”

(Last Updated: 1 September 2006)

### Chapter 7

**Q7.1** Note that  $\mu_1 = m_1/(m_s + m_2)$  and  $\mu_2 = m_2/(m_s + m_1)$  where  $m_s$  is the mass of the star. See error listing.

**Q7.2** The precession rate of Saturn due to Jupiter is  $2.6 \times 10^{-3} \text{ }^\circ\text{y}^{-1}$  using this method. The formula for  $g_-$  given in Q7.1 is identical to that derived in Q6.3. The precession rate of Venus due to Earth is  $6.9 \times 10^{-4} \text{ }^\circ\text{y}^{-1}$  using this method. The precession rate of the lunar orbit due to the Sun is  $5.53 \times 10^{-2} \text{ }^\circ\text{d}^{-1}$  using this method; this gives a precessional period of 17.8 y. The precession rate due to the Earth’s  $J_2$  is  $5.88 \times 10^{-6} \text{ }^\circ\text{d}^{-1}$ ; this is much smaller than the solar effect.

**Q7.3** The rate is  $\dot{\varpi} = \frac{1}{4}(m'/M)n\alpha \left\{ (2\alpha D + \alpha^2 D^2)b_{1/2}^{(0)} + (e'/e)(2 - 2\alpha D - \alpha^2 D^2)b_{1/2}^{(1)} \cos(\varpi' - \varpi) \right\}$ .  
 (i) When  $\varpi' = \varpi$ ,  $\dot{\varpi} = 1.447^\circ/\text{century}$ . (ii) When  $\varpi' = \varpi + 180^\circ$ ,  $\dot{\varpi} = 2.220^\circ/\text{century}$ .

**Q7.4** Using the secular theory for Jupiter and Saturn alone gives  $e_{\text{forced}} = 0.0350$ ,  $\varpi_{\text{forced}} = 9.40^\circ$ ,  $e_{\text{free}} = 0.0452$ ,  $\varpi_{\text{forced}} = 122.54^\circ$ ,  $I_{\text{forced}} = 1.150^\circ$ ,  $\Omega_{\text{forced}} = 96.55^\circ$ ,  $I_{\text{free}} = 2.091$ ,  $\Omega_{\text{forced}} = 300.34^\circ$ . Using Brouwer & van Woerkom’s secular theory gives  $e_{\text{forced}} = 0.0371$ ,  $\varpi_{\text{forced}} = 6.28^\circ$ ,  $e_{\text{free}} = 0.0479$ ,  $\varpi_{\text{forced}} = 123.85^\circ$ ,  $I_{\text{forced}} = 1.149^\circ$ ,  $\Omega_{\text{forced}} = 96.02^\circ$ ,  $I_{\text{free}} = 2.086$ ,  $\Omega_{\text{forced}} = 300.08^\circ$ .

**Q7.5** For a density of  $1.2 \text{ g cm}^{-3}$  the minimum separation varies from 57.8 km for a zero mass F ring, to 162.2 km for an F ring mass equal to three times that of Prometheus. The separation is less than 70 km when  $0 \leq m < 0.4$  where  $m$  is measured in units of a Prometheus mass. For a density of  $0.6 \text{ g cm}^{-3}$  the minimum separation varies from 79.5 km for a zero mass F ring to 132.5 km for an F ring mass equal to three times that of Prometheus. The separation is never less than 70 km.

**Q7.6** The eccentricity–pericentre eigenfrequencies,  $g_i$  (in degrees per day) and the associated locations in semi-major axes (in km) where  $A = g_i$  are  $g_1 = 1.00184$  (187488, 237296, 238739, 292761, 296549, 374945, 379850, 523206, 530872, 1186250, 1257440, 1480710, 1481490, 3556240, 3566360),  $g_2 = 0.417727$  (241742, 290962, 298218, 373313, 381449, 520975, 533089, 1166710, 1276970, 1480490, 1481710, 3553470, 3569130),  $g_3 = 0.198099$  (305715, 370258, 384213, 517838, 536160, 1141710, 1301890, 1480190, 1482010, 3549920, 3572680),  $g_4 = 0.0842694$  (397518, 510745, 542585, 1098550, 1344560, 1479590, 1482600, 3543850),  $g_5 = 0.0275128$  (574249, 1000210, 1436770, 1476800, 1485110, 353070, 3591890),  $g_6 = 0.0188353$  (618933, 944487, 1491350, 3524270, 3598320),  $g_7 = 0.00137212$  (2145640, 3414930, 3704710),  $g_8 = 0.000152186$  (4161030). The inclination–node eigenfrequencies,  $f_i$  (in degrees per day) and the associated locations in semi-major axes (in km) where  $B = f_i$  are  $f_1 = -0.999287$  (187491, 237294, 238740, 292758, 296552, 374942, 379853, 523201, 530877, 1186210, 1257490, 1480710, 1481490, 3556240, 3566360),  $f_2 = -0.417079$  (241745, 290958, 298222, 373309, 381453, 520970, 533094, 1166670, 1277010, 1480490, 1481710, 3553460, 3569140),  $f_3 = -0.197899$  (305721, 370253, 384217, 517833, 536165, 1141670, 1301930, 1480190, 1482010, 3549920, 3572680),  $f_4 = -0.0842182$  (397524, 510739, 542591, 1098510, 1344590, 1479590, 1482600, 3543840),  $f_5 = -0.0275047$  (574257, 1000170, 1436800, 1476800, 1485110, 353070, 35918900),  $f_6 = -0.0188356$  (618899, 944493, 1491350, 3524270, 3598320),  $f_7 = -0.00137098$  (2146000, 3414860, 3704780),  $f_8 = -0.00015199$  (4161600).