Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 1 September 2006)

Chapter 10

Q10.1 The exact resonance is located at 136792 km giving a wave of amplitude of 2.0 km.

Q10.2 The Prometheus 13:12 ILR is at 132196 km; the first observed feature is just exterior to this location.

Q10.3 Under this model the mass of Pan is 1.96×10^{17} kg and its density is an unbelievable 46.9 g cm^{-3}! (This paradaox can be resolved by realising that the width of the cleared gap is determined by the shepherding mechanism; see Cuzzi & Scargle 1985.) Using this large mass gives $p = 258$ for overlap and this occurs at a separation in semi-major axis of 345 km. The maximum radial width of a horseshoe is twice the distance to the L_1 point or ~ 130 km; the observed ring is much narrower than this (see Fig. 10.21a). For the last part of the question think about resonance overlap.

Q10.4 The ratio of the widths is $W_{\rm cr}/W_{\rm Lr} = \sqrt{2}\left(e'(j-1)[-1+2j+\alpha D]b_{1/2}^{(j-1)}/[2j+\alpha D]b_{1/2}^{(j)}\right)^{1/2}$.

Q10.5 The locations and widths of the Cordelia OLRs in the stated range are 26:27 at 51014.8 km (width 1.689 km), 25:26 at 51065.1 km (width 1.691 km), 24:25 at 51119.5 km (width 1.693 km), 23:24 at 51178.6 km (width 1.695 km) and 22:23 at 51243.0 km (width 1.697 km). There is only one Ophelia ILR in the range; it is the 15:14 at 51176.6 km (width 2.300 km). The 24:25 Cordelia OLR produces a wave of amplitude 0.34 km. The 15:14 Ophelia ILR produces a wave of amplitude 0.24 km. *Note*: When dealing with the OLRs in this question you cannot make use of the formulae given in Eqs. (10.21)–(10.23) because these are only valid for ILRs. However, similar formulae can be derived. These give $e_{\rm f}' = |n' f_{\rm d}\,(m/m_{\rm c})\,/\,[(j-1)n - jn']|$, $a'e_{\rm f}' = 2a'^2\,|f_{\rm d}|\,(m/m_{\rm c})\,/\,(3j\,|a'-a_{\rm res}'|)$ and $W' = 4a'\left\{2\,|f_{\rm d}|\,(m/m_{\rm c})\,/(3j)\right\}^{1/2}$ where $f_{\rm d} = \frac{1}{2}\,[-1+2j+\alpha D]\,b_{1/2}^{(j-1)}$.

Q10.6 Here we have assumed that the density of Galatea is 1.2 g cm^{-3}. (a) The 42:43 CIR is at 62929.5 km. (b) The width of the 42:43 CIR is 0.500 km. (c) This has to be done very carefully! See section II.A of the paper by Horanyi & Porco (1993) for a complete explanation and numerical evaluation. By taking account of the variation of the mean longitude at epoch, Horanyi & Porco found that the location of the resonances shifted outwards by 0.2 km. (d) The 42:43 OLR is at 62927.8 km. (e) Using the formulae given in the answer to Q10.5 above, the radial amplitude of the wave should be 31 km. Compare this with the full width of the OLR (29 km) and the width of the CIR (0.5 km)! (f) Check out fig. 2 in the paper by Goldreich *et al.* (1986) for an idea of how this might look. *Note*: The numbers used in this question (apart from the assumed density of Galatea) were taken from Appendix A and are not identical to those used by Porco (1991). Furthermore, the mean motion of Galatea and hence the location of the resonances have been revised by Sicardy *et al.* in *Nature* **400**, 731–733 (1999).