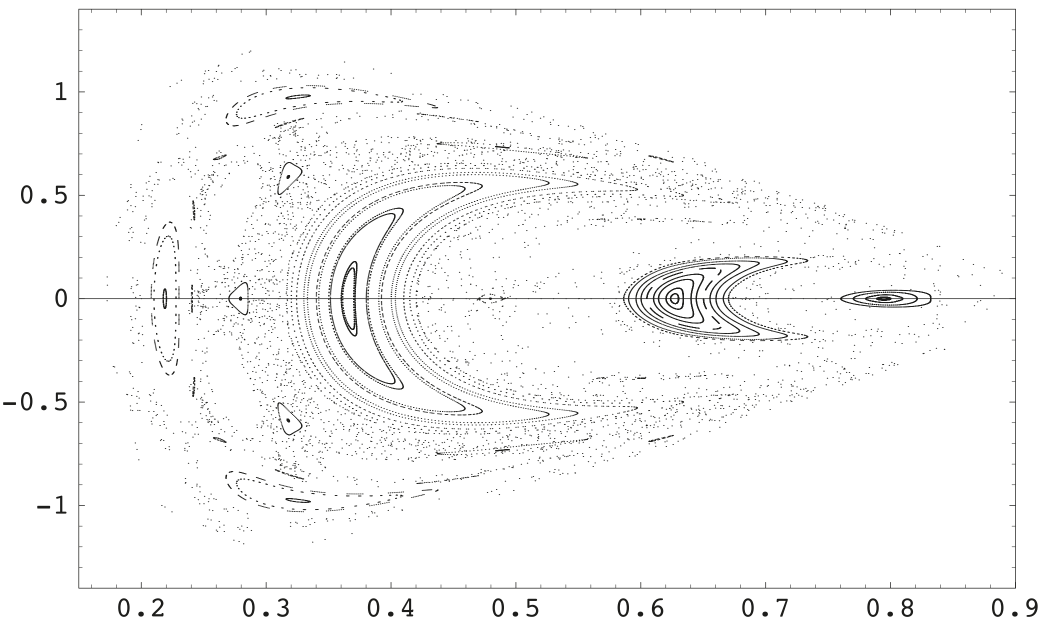
Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 28 January 2021)

Chapter 9

Q9.1 For $\lambda_0 = 293°$, $\Delta a = +2.989$, $\Delta e = +0.3340$, $\Delta\varpi = -238.3°$. For $\lambda_0 = 293.3°$, $\Delta a = +4.508$, $\Delta e = +0.4111$, $\Delta\varpi = -223.9°$. By varying the initial mean longitude in steps of 0.1° from 0 to 360° we get $\Delta a_{\max} = +4.508$ for $\lambda_0 = 293.3°$ (one of the two starting conditions above) and $\Delta a_{\min} = -0.1836$ for $\lambda_0 = 334.8°$.

Q9.2 The surface of section plot should look something like the figure below. In the plot the horizontal axis is x and the vertical axis is $\dot{x}$. There are also some points with negative values of x but these have been excluded for clarity. The islands at $x = 0.218$, 0.279, 0.365, 0.479 and 0.626 are associated with the 5:2, 9:4, 2:1, 7:4 and 3:2 resonances. Note that because these are all of odd order, they have an odd number of islands and therefore, because of symmetry, they will have islands on the x-axis. The apparent island close to $x = 0.79$ is actually associated with a periodic orbit of the first kind (see Winter & Murray, 1994a).



Q9.3 Taking $\mu = 1/1048.672$ (from values of the solar and jovian masses given in Tables A.1 and A.2) and using the absolute value of the difference in the eccentricities for the first 3500 Jupiter periods gives a maximum Lyapounov characteristic exponent for the eccentricity of 0.00399 per Jupiter period. Note that the divergence is minute at first and then becomes larger. Because there is no renormalisation the later data values are not good to use in the calculation of the Lyapounov characteristic exponent.

Q9.4 Taking $\mu = 1/1048.672$ (from values of the solar and jovian masses given in Tables A.1 and A.2) and starting values in the stated range gives maximum eccentricities above 0.3 for the following values of a and e: (i) $a = 0.470-0.476$ with $e = 0.3$; (ii) $a = 0.478$ with $e = 0.27-0.30$; (iii) $a = 0.479$ with $e = 0.22-0.26$ and $e = 0.30$; (iv) $a = 0.480$ with $e = 0.16-0.17$ and $e = 0.30$; (v) $a = 0.481$ with $e = 0.04$, $e = 0.12-0.18$ and $e = 0.29-0.30$; (vi) $a = 0.482$ with $e = 0.16-0.18$, $e = 0.21-0.24$ and $e = 0.26-0.30$; (vii) $a = 0.483$ with $e = 0.21-0.30$; (viii) $a = 0.484$ with $e = 0.25-0.30$; (ix) $a = 0.485$ with $e = 0.28-0.30$; (x) $e = 0.29-0.30$ with $a = 0.486-0.488$; (xi) $e = 0.30$ with $a = 0.489-0.490$. Note that the chaotic nature of some of these orbits means that your results could have small differences from those quoted above.

Q9.5 The largest semi-major axis before the condition becomes satisfied is $a = 0.773$; i.e. the first value for which $e > \Delta a/a'$ is $a = 0.774$. Using the overlap criterion given in Eq. (9.148), the predicted value is $a = 0.822$. Note that the value of $\Delta a/a'$ from the encounter map is larger than that predicted; the same over-estimate is seen in Fig. 9.23 for a range of mass ratios.

Q9.6 The maximum absolute differences in longitude for each planet for $\Delta t = 1$d and $\Delta t = 10$d, respectively, are as follows. Mercury: 0.0754°, 0.0730°. Venus: 0.191°, 0.193°. Earth: 0.0358°, 0.0331°. Mars: 0.183°, 0.183°. Jupiter: 0.377°, 0.377°. Saturn: 4.41°, 4.41°. Uranus: 0.511°, 0.511°. Neptune: 0.537°, 0.537°. Pluto: 1.022°, 1.022°.