Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 1 September 2006)

Chapter 6

Q6.1 $8n' - 3n = -0.00789237\,^\circ\mathrm{d}^{-1}$. If we write a general argument as $\varphi = 8\lambda' - 3\lambda + \varphi_i$ then the φ_i are $\varphi_1 = -5\varpi$, $\varphi_2 = -\varpi' - 4\varpi$, $\varphi_3 = -2\varpi' - 3\varpi$, $\varphi_4 = -3\varpi' - 2\varpi$, $\varphi_5 = -4\varpi' - \varpi$, $\varphi_6 = -5\varpi'$, $\varphi_7 = -3\varpi - 2\Omega$, $\varphi_8 = -\varpi' - 2\varpi - 2\Omega$, $\varphi_9 = -2\varpi' - \varpi - 2\Omega$, $\varphi_{10} = -3\varpi' - 2\Omega$, $\varphi_{11} = -\varpi - 4\Omega$, $\varphi_{12} = -\varpi' - 4\Omega$, $\varphi_{13} = -3\varpi - \Omega' - \Omega$, $\varphi_{14} = -\varpi' - 2\varpi - \Omega' - \Omega$, $\varphi_{15} = -2\varpi' - \varpi - \Omega' - \Omega$, $\varphi_{16} = -3\varpi' - \Omega' - \Omega$, $\varphi_{17} = -\varpi - \Omega' - 3\Omega$, $\varphi_{18} = -\varpi' - \Omega' - 3\Omega$, $\varphi_{19} = -3\varpi - 2\Omega'$, $\varphi_{20} = -\varpi' - 2\varpi - 2\Omega'$, $\varphi_{21} = -2\varpi' - \varpi - 2\Omega'$, $\varphi_{22} = -3\varpi' - 2\Omega'$, $\varphi_{23} = -\varpi - 2\Omega' - 2\Omega$, $\varphi_{24} = -\varpi' - 2\Omega' - 2\Omega$, $\varphi_{25} = -\varpi - 3\Omega' - \Omega$, $\varphi_{26} = -\varpi' - 3\Omega' - \Omega$, $\varphi_{27} = -\varpi - 4\Omega'$, $\varphi_{28} = -\varpi' - 4\Omega'$. The term associated with the argument $\varphi = 8\lambda' - 3\lambda - \varpi' - 2\varpi - \Omega' - \Omega$ is $\mathcal{G}(m'/a')e^2e'ss'\frac{1}{16}\left\{-1488\alpha b_{3/2}^{(6)} - 433\alpha^2 \mathrm{d}b_{3/2}^{(6)}/\mathrm{d}\alpha - 38\alpha^3\mathrm{d}^2 b_{3/2}^{(6)}/\mathrm{d}\alpha^2 - \alpha^4\mathrm{d}^3 b_{3/2}^{(6)}/\mathrm{d}\alpha^3\right\}$. The Laplace coefficients are $\alpha b_{3/2}^{(6)} = 0.0942442$, $\alpha^2\mathrm{d}b_{3/2}^{(6)}/\mathrm{d}\alpha = 0.677156$, $\alpha^3\mathrm{d}^2 b_{3/2}^{(6)}/\mathrm{d}\alpha^2 = 4.49721$ and $\alpha^4\mathrm{d}^3 b_{3/2}^{(6)}/\mathrm{d}\alpha^3 = 28.5378$. The smallest integers for which the condition is satisfied are $p = 10$ and $q = 17$.

Q6.2 The differential equation for G is $(y-1)y\,\mathrm{d}^2G/\mathrm{d}y^2 + [(2s+j+1)y - 2s]\,\mathrm{d}G/\mathrm{d}y + s(s+j)G = 0$. Taking $s = \frac{3}{2}$ (see error listing), substituting the solution for G in this differential equation and equating coefficients of y^{-2} and y^{-1} gives $A_1 = \frac{1}{4}(2j-1)A_0$ and $B_0 = \frac{1}{8}(2j+1)A_1$ respectively. Taking $l = k$ and $l = k+1$ and equating coefficients of $y^k \ln y$ gives $B_{k+1} = \frac{1}{4}B_\mathrm{k}(2k+3)(2k+2j+3)/[(k+1)(k+3)]$. Taking $l = k-2$, $k-1$, k and $k+1$ and equating coefficients of y^{k-2} gives $A_{k+1} = [-2kB_{k-1} + (2k+j-1)B_{k-2} + \frac{1}{4}(1-2k)(1-2k-2j)A_k]/[(k+1)(k-1)]$. Although A_0 and A_2, on which all the remaining A_l and B_l depend, are not defined, they can be calculated numerically. Consider G as a function of y, A_0 and A_2. We can isolate A_0 and A_2 as factors by noting that $G(y; A_0, A_2) = A_0\,G(y;1,0) + A_2\,G(y;0,1)$. Now, since $F(x) = G(y)$ where $y = 1-x$, we can evaluate $F(x)$ at two arbitrary values of $x = 1-y$ giving the two simultaneous equations $F_1 = F(1-y_1) = G(y_1) = A_0\,G(y_1;1,0) + A_2\,G(y_1;0,1)$ and $F_2 = F(1-y_2) = G(y_2) = A_0\,G(y_2;1,0) + A_2\,G(y_2;0,1)$. Because we know the form of F we are left with two linear equations in two unknowns, A_0 and A_2. The choice of y_1 and y_2 is arbitrary provided they are different positive values less than 1. The value of $b_{3/2}^{(2)}(0.999)$ is 636930.0087516. This sort of precision can be achieved with $l < 10$ in the series for G.

Q6.3 $\dot\varpi_\mathrm{S} = -\dot\Omega_\mathrm{S} \approx (3/4)\,(m_\mathrm{J}/M)\,(a_\mathrm{J}/a_\mathrm{S})^2\,n_\mathrm{s}$. This gives a pericentre precession period of 137000 y. Note that this is off by a factor 2.5 but the right order of magnitude. It would have been a better approximation if $a_\mathrm{J} \ll a_\mathrm{S}$.

Q6.4 $\mathrm{d}I/\mathrm{d}t = -\left(3nJ_3R^3e\right)\cos I\left((5/4)\sin^2 I - 1\right)\cos\omega/\left(2a^3(1-e^2)^3\right)$. The expression for $\mathrm{d}\omega/\mathrm{d}t$ is not given in Sect. 6.11 (see error listing) so you should use: $\mathrm{d}\omega/\mathrm{d}t = \left(3nJ_2R^2\right)\left(1-(5/4)\sin^2 I\right) / \left(a^2(1-e^2)^2\right)$. The approximate variation in I is $\pm\,(J_3Re)\,/\,\left(2J_2a(1-e^2)\right)$.

Q6.5 (a) $n = \left(\mathcal{G}M/a^3\right)^{1/2}\left\{1 + 3h^2/(c^2a^2)\right\}^{1/2}$ and $\kappa = \left(\mathcal{G}M/a^3\right)^{1/2}\left\{1 - 3h^2/(c^2a^2)\right\}^{1/2}$. (b) For Earth $\dot\varpi_\mathrm{GR} = 0.038$ arcsec y^{-1}. For Mercury $\dot\varpi_\mathrm{GR} = 0.41$ arcsec y^{-1}.

Q6.6 (a) $a_\mathrm{c} = \left\{2J_2\,(M_\mathrm{p}/M_\mathrm{Sun})\,R^2a_\mathrm{p}^3\cos I/\cos\beta\right\}^{1/5}$. (b) For Earth, Saturn and Uranus $a_\mathrm{c}/R = 9.82$, 42.5 and 77.6, respectively. (c) Think about the important plane in each case. (d) Here we are calculating the precession due to J_2 alone. Moon: $\dot\Omega = -5.9\times10^{-6}\,^\circ\mathrm{d}^{-1}$; $T = 6.1\times10^7$ d. Mimas: $\dot\Omega = -0.99^\circ\mathrm{d}^{-1}$; $T = 365$ d. Titan: $\dot\Omega = -0.0013^\circ\mathrm{d}^{-1}$; $T = 2.7\times10^5$ d. Miranda: $\dot\Omega = -0.052^\circ\mathrm{d}^{-1}$; $T = 6.9\times10^3$ d. Oberon: $\dot\Omega = -2.7\times10^{-4}\,^\circ\mathrm{d}^{-1}$; $T = 1.3\times10^6$ d.