Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 1 September 2006)

Chapter 5

Q5.1 (a) For Io: $a - c = 15.6\text{km} = 0.00856 R_{\text{Io}}$. (b) For the Moon in its current orbit: $a - c = 0.0658\text{km} = 0.0000379 R_{\text{Moon}}$; this is undetectable. (c) For the Moon at 10 Earth radii: $a - c = 14.4\text{km} = 0.00830 R_{\text{Moon}}$; for the Earth with a 10 h rotation period and $h_2 = 1.94$: $\epsilon = 0.0192$ and $J_2 = 0.00619$. (d) Taking the spin period to be 58.65 days (see error listing) you should get $\epsilon = 1.3 \times 10^{-6}$; this corresponds to $a - c = 0.003$ km. (e) In this part you could use the mean density for Pluto given in Table A.4. For Pluto $a - c = 1.3$ km; for Charon $a - c = 0.7$ km.

Q5.2 (a) The total mass is 1.47×10^{22} kg giving a mean density of $\langle \rho \rangle = 2.08$ g cm^{-3}. See Sect. 2.7 for a clue as to how to measure the individual masses and densities. (b) $\tau_{\text{P}} \approx 7 \times 10^6$ y. (c) Note that for smaller bodies $k_2 \propto R^2$ and so $\tau_{\text{C}}/\tau_{\text{P}} \approx (R_{\text{C}}/R_{\text{P}})^4$ is more realistic. (d) The intial semi-major axis and orbital period would have been $a = 14620$ km and $P = 4.103$ d, respectively.

Q5.3 This is a fairly simple task using the outline in Sect. 5.3.

Q5.4 The critical eccentricity is $e_{\text{crit}} = 0.2793$.

Q5.5 Measuring along the $\dot{\theta}/n = 1$ line, the maximum variation is reduced from 180° to 76°.

Q5.6 For $p = -\frac{1}{2}$, $\left(\dot{\theta}/n\right)_{\text{max/min}} = -\frac{1}{2} \pm \sqrt{e^3/48}$. For $p = -1$, $\left(\dot{\theta}/n\right)_{\text{max/min}} = -1 \pm \sqrt{e^4/24}$. For fixed e intersection of the "islands" of these resonances occurs for $\alpha = \frac{1}{2}\left\{\sqrt{e^3/48} + \sqrt{e^4/24}\right\}^{-1}$. The equation for e as a function of α is transcendental in e and so has to be solved numerically. Based on the equation for α as a function of e, are the islands ever likely to intersect?